

Inertial range turbulence in kinetic plasmas

Gregory G. Howes

601 Campbell Hall, Department of Astronomy, University of California, Berkeley, CA 94720, USA.

(Dated: February 2, 2008)

The transfer of turbulent energy through an inertial range from the driving scale to dissipative scales in a kinetic plasma followed by the conversion of this energy into heat is a fundamental plasma physics process. A theoretical foundation for the study of this process is constructed, but the details of the kinetic cascade are not well understood. Several important properties are identified: (a) the conservation of a generalized energy by the cascade; (b) the need for collisions to increase entropy and realize irreversible plasma heating; and (c) the key role played by the entropy cascade—a dual cascade of energy to small scales in both physical and velocity space—to convert ultimately the turbulent energy into heat. A strategy for nonlinear numerical simulations of kinetic turbulence is outlined. Initial numerical results are consistent with the operation of the entropy cascade. Inertial range turbulence arises in a broad range of space and astrophysical plasmas and may play an important role in the thermalization of fusion energy in burning plasmas.

PACS numbers: 52.30.Gz—52.35.Ra

Keywords: Gyrokinetics—plasma turbulence

I. INTRODUCTION

In a warm, magnetized kinetic plasma, the dissipation of low-frequency turbulent fluctuations by kinetic mechanisms occurs most strongly at length scales of order the plasma particle Larmor radii. If turbulence is driven in the plasma at a much larger scale, an inertial range develops in which the injected energy is cascaded by nonlinear interactions to the small scales at which the turbulence can be effectively dissipated. This transfer of turbulent energy through an inertial range from the driving scale to dissipative scales followed by the conversion of this energy into heat is a fundamental kinetic plasma physics process. In this paper, I lay out a theoretical foundation on which to build our knowledge of *inertial range turbulence in kinetic plasmas* and outline a practical strategy for the nonlinear numerical modeling of this process.

The study of turbulence in kinetic plasmas is not a new endeavor; in magnetic fusion community, understanding the effect of turbulence on the transport properties of the plasma is one of the primary goals of the numerical modeling effort. But the turbulence most often investigated in laboratory plasmas is driven by microinstabilities arising from gradients in the background plasma—for example, ion temperature gradient (ITG) and electron temperature gradient (ETG) instabilities [1, 2, 3, 4, 5, 6, 7, 8, 9]. These instabilities inject energy into the plasma at the scale of the particle Larmor radius; the resulting turbulence does not develop an inertial range because the driving and kinetic dissipation scales are of roughly the same order. Inertial range turbulence in kinetic plasmas, therefore, represents a fundamentally different process than the kinetic turbulence traditionally studied by the fusion community.

The turbulence in most astrophysical contexts, on the other hand, is typically driven at scales much larger than the Larmor radius and gives rise to a turbulent cascade through an inertial range to the dissipative scales at

which the energy of turbulent fluctuations is ultimately converted into particle thermal energy. Before constructing the detailed theoretical foundation for the study of this process, presented in Sec II, I will provide here a simplified blueprint of the physical mechanisms guiding the flow of energy. It is important to emphasize that many of the mechanisms described below are not well understood. Here I aim only to paint the overall picture in broad strokes, deferring discussion of the many uncertainties to Sec II.

At a scale larger than the particle mean free path—a collisional scale for which magnetohydrodynamics (MHD) provides an adequate description—the plasma is stirred by some external mechanism, driving an assortment of MHD Alfvén, fast, slow, and entropy mode fluctuations in the plasma. At this driving scale, these modes are undamped; thus, a turbulent cascade develops nonlinearly to transfer the fluctuation energy to smaller scales. The compressive modes become damped as the cascade reaches scales of order or smaller than the collisional mean free path, but the Alfvénic cascade continues undamped down to the scale of the ion Larmor radius. At this kinetic scale, the electromagnetic fluctuations may be damped collisionlessly by the Landau resonance with the ions. In the absence of collisions, this process conserves a generalized energy—the free energy removed from the electromagnetic fluctuations generates nonthermal structure in velocity space of the ion distribution function. The remaining electromagnetic fluctuation energy continues to cascade below the scale of the ion Larmor radius as a kinetic Alfvén wave cascade. Upon reaching the scale of the electron Larmor radius, the electromagnetic fluctuations of the kinetic Alfvén wave cascade are completely damped via the Landau resonance with the electrons; again, a generalized energy is conserved in this process, leading to the creation of nonthermal structure in velocity space of the electrons. But the damping of the electromagnetic fluctuations and conse-

quent generation of structure in velocity space does not correspond to heating; irreversible heating requires an increase in entropy that can only be achieved by collisions. The thermalization of the turbulent energy by collisions is ultimately achieved thanks to a cascade to small scales in velocity space of the particle distribution functions—an *entropy cascade*. The entropy cascade drives the distribution function structure in velocity space to scales small enough that even weak collisions are sufficient to smooth out that structure towards the Maxwellian, causing entropy to increase—this is the final step in the conversion of the energy of the turbulent fluctuations to thermal energy of the plasma particles.

The entire process described above is the kinetic generalization of the familiar cascade of energy in a fluid turbulent system; this fundamental kinetic plasma physics process, encompassing the dual cascade in both physical and velocity space, is referred to as the *kinetic cascade* of the generalized energy [10]. Neither the detailed interactions of the kinetic cascade nor its implications for the flow of energy in turbulent systems are well understood; a theoretical foundation for its study is laid out in Sec II. Non-linear numerical simulations of the kinetic cascade occurring in inertial range turbulence are expected to play a leading role in shedding light on the complex physical mechanisms involved—a strategy for the numerical approach to this problem is presented in Sec III and initial numerical results are discussed in Sec IV. Astrophysical and laboratory environments in which the kinetic cascade of inertial range turbulence may play an important role are identified in Sec V.

II. THEORETICAL FOUNDATION

Although inertial range turbulence plays an important role in the evolution of many kinetic plasmas, it has not yet attracted great attention within the scientific community. The physics underlying this intersection of kinetic plasmas and turbulence is fascinating yet remains poorly understood. This paper aims to construct a foundation upon which a thorough knowledge of the varied mechanisms involved may be built. I present here a view of the overall context in which the kinetic cascade operates and aim to emphasize the uncertainty underlying some aspects of this view.

I begin with the assumption that the turbulence is driven at a scale L much larger than the ion collisional mean free path, $L \gg \lambda_{\text{mfp}_i}$. The dynamics at the driving scale is therefore collisionally dominated, so the single fluid magnetohydrodynamic (MHD) theory provides an adequate description of the plasma dynamics at all parallel scales l_{\parallel} satisfying $l_{\parallel} \gg \lambda_{\text{mfp}_i}$ (where \perp and \parallel in this paper are relative to the direction of the local mean magnetic field)[82]. The assumption $L \gg \lambda_{\text{mfp}_i}$ will not be true for all environments in which inertial range, kinetic turbulence arises; the implications of relaxing this assumption are discussed in Sec III. Based on the fact

that the damping of all linear wave modes at the MHD scales $l_{\parallel} \gg \lambda_{\text{mfp}_i}$ is negligible, the damping of the turbulent fluctuations is likewise expected to be negligible; an inertial range will then develop to cascade the turbulent energy via nonlinear interactions to ever smaller scales. This cascade will continue down to the transition from fluid to kinetic behavior at a scale $l_{\parallel} \sim \lambda_{\text{mfp}_i}$. At this point, a fluid treatment of the turbulence is no longer adequate—instead a kinetic theory must be used to describe the dynamics. The nature of the kinetic continuation of the inertial range to scales $l_{\parallel} < \lambda_{\text{mfp}_i}$ depends critically on the character of the MHD turbulent fluctuations reaching this transition.

Here I review five key concepts underpinning the theory of MHD turbulence before applying them to construct a model for the fluid portion of the turbulent inertial range.

1. The Kolmogorov Hypothesis: the nonlinear energy transfer is constant through all scales within the inertial range, defined as the extent of scales influenced neither by the energy injection mechanism nor by dissipation; at a given scale, the energy transfer is spectrally local, with a rate determined only by the turbulent conditions at that scale [11].
2. The Kraichnan Hypothesis: even in a magnetized plasma with no mean field, the magnetic field of large-scale fluctuations behaves effectively as a mean field for fluctuations on smaller scales [12].
3. Colliding Wavepackets: as is apparent when the equations of incompressible MHD are cast into Elsässer form, only Alfvén wave packets traveling in opposite directions along the mean magnetic field interact nonlinearly [12, 13].
4. Critical Balance: as strong MHD turbulence cascades to small scale, it maintains a state of balance between the (parallel) linear propagation and (perpendicular) nonlinear interaction timescales [14, 15].
5. Decoupling of Fast Wave Dynamics: theory and simulations of compressible MHD turbulence suggest that the isotropic cascade of fast MHD wave turbulence decouples from the dynamics of the Alfvén, slow, and entropy modes [16, 17].

A. Weak MHD Turbulence

Adopting the Kolmogorov and Kraichnan hypotheses, the interaction of oppositely directed, small-amplitude Alfvén wave packets along the mean (or large-scale) magnetic field in incompressible MHD is considered. If it is assumed (a) that the nonlinear interaction between two colliding wavepackets is weak—and therefore that a single wave packet must undergo many collisions before

it has been distorted enough to have effectively transferred its energy to smaller scale—and (b) that the energy is transferred to smaller scales *isotropically*, an inertial range one-dimensional energy spectrum that scales with wavenumber as $k^{-3/2}$ is predicted. A spectrum with this scaling is often referred to as the Iroshnikov-Kraichnan spectrum [12, 13].

However, evidence from laboratory plasmas [18, 19, 20], the solar wind [21] and numerical simulations [22, 23] suggests that the MHD turbulent cascade is not isotropic but preferentially transfers energy to small scales perpendicular to the mean magnetic field. Taking the observed anisotropy into account, anisotropic theories of MHD turbulence have been proposed and refined [14, 20, 22, 24, 25, 26, 27, 28], and have led to the emergence of a mature theory for weak incompressible MHD turbulence [29, 30, 31]. The predicted one-dimensional energy spectrum for weak incompressible MHD turbulence scales as k_{\perp}^{-2} ; the cascade is anisotropic, with all energy cascading to higher perpendicular wavenumbers and none to higher parallel wavenumbers. Unlike the isotropic Iroshnikov-Kraichnan cascade, in which the nonlinear interactions weaken as the turbulence progresses to smaller scale, this anisotropic theory predicts that, as the turbulence cascades to higher perpendicular wavenumber, the nonlinear interactions strengthen, leading eventually to the inevitable violation of the assumption of weak nonlinear interactions [25, 28, 30, 31]. Thus arises the most important implication of the weak turbulence theory: given a cascade through a sufficiently broad range of scales, weak incompressible MHD turbulence will inevitably transition to a state of strong turbulence.

B. Strong MHD Turbulence

Below the perpendicular scale where the perturbative treatment fails, the resulting state of strong incompressible MHD turbulence is treated phenomenologically by assuming that the turbulence maintains a state of critical balance, $\omega \sim \omega_{nl}$, as it cascades to smaller scales. For a linear Alfvén wave frequency, $\omega = k_{\parallel} v_A$, and a nonlinear frequency determined by the perpendicular dynamics, $\omega_{nl} \sim k_{\perp} v_{\perp}$, theory predicts a one-dimensional kinetic energy spectrum $E_k(k_{\perp}) \propto k_{\perp}^{-5/3}$ and a scale-dependent anisotropy $k_{\parallel} \propto k_{\perp}^{2/3}$. Although numerical simulations of strong MHD turbulence with a dynamically strong mean field appear to support the scale-dependent anisotropy $k_{\parallel} \propto k_{\perp}^{2/3}$ [32, 33], these same simulations routinely produce one-dimensional energy spectra that appear to scale as $k_{\perp}^{-3/2}$ rather than $k_{\perp}^{-5/3}$ [33, 34]. A modification of the theory to take into account a proposed polarization alignment [35] may shed light on this discrepancy, but the matter of strong MHD turbulence remains controversial. It is also worthwhile noting here that the predicted transition from weak to strong MHD turbulence has only

recently been numerically reproduced [36].

In spite of the uncertainties in the theory for the strong incompressible MHD turbulence, the combination of key concepts (1)–(4) in Sec II leads to the following general prediction: in any magnetized, incompressible plasma, whether or not a strong mean field exists at the large scale, at sufficiently small scales a state of strong MHD turbulence will arise. Therefore, given the assumption of sufficiently large driving scale $L \gg \lambda_{\text{mfp}i}$, at the transition from fluid to kinetic behavior $l_{\parallel} \sim \lambda_{\text{mfp}i}$, the turbulent fluctuations will be dominantly perpendicular with $k_{\parallel} \ll k_{\perp}$ and the fluctuation amplitudes will be small compared to the mean field, $\delta B_{\perp} \ll B_0$. The kinetic description of the continued turbulent cascade to smaller scales is strongly influenced by these predicted characteristics of the turbulence.

C. Compressible MHD Turbulence

The discussion thus far has been limited to incompressible MHD. In the case of compressible MHD, an arbitrary stirring mechanism may inject energy into the compressive modes of the plasma—the fast, slow, and entropy modes. A weak turbulence treatment of compressible MHD for fast and Alfvén waves suggests that only a small amount of energy is transferred from fast waves to Alfvén waves at large k_{\parallel} [37]. Numerical simulations of strong turbulence in compressible MHD demonstrate an isotropic cascade of fast waves that scales as $k^{-7/2}$ and suggest that this cascade of fast wave energy is decoupled from the Alfvén and slow wave cascades [17]. In the region of wavevector space where the energy of the Alfvén wave cascade is concentrated, $k_{\parallel} \ll k_{\perp}$, the fast wave frequencies $\omega \sim k_{\perp} \sqrt{c_s^2 + v_A^2}$, where $c_s^2 = \gamma p / (n_i m_i)$ is the sound speed, are much higher than the Alfvén wave frequencies $\omega = k_{\parallel} v_A$, so the decoupling of these cascades is not surprising. The cascade of the slow waves and entropy modes is slaved to the Alfvén wave cascade [10, 33] and is seen to mimic the anisotropic cascade of the Alfvén waves [17, 33]. Although, from these arguments, the compressive modes of the plasma turbulence are not expected to significantly alter the behavior of the Alfvén wave cascade as determined by incompressible MHD turbulence theory, this topic remains an open area of research.

D. Transition to Kinetic Turbulence at $l_{\parallel} \sim \lambda_{\text{mfp}i}$

As the inertial range turbulent cascade enters the regime of weak collisionality at $l_{\parallel} \sim \lambda_{\text{mfp}i}$, the nature of the fluctuations at this transition motivates a particular choice for the kinetic description of the dynamics of the cascade to yet smaller scales. The theory of MHD turbulence described above predicts the following turbulent state at the transition: isotropic fast wave fluctu-

ations and anisotropic Alfvén, slow, and entropy mode fluctuations with $k_{\parallel} \ll k_{\perp}$. Because both the energy of the fluctuations decreases with increasing wavenumber ($\propto k^{-7/2}$ for fast waves and $\propto k_{\perp}^{-5/3}$ for Alfvén, slow, and entropy modes) and the driving scale is assumed large $L \gg \lambda_{\text{mfp}i}$, the amplitude of the magnetic field fluctuations at this scale will be small compared to the local mean field, $\delta B_{\perp} \ll B_0$. Although strong nonlinear interactions dominate the turbulence—leading to the nonlinear cascade of the energy of a wave on a timescale of order the wave period—the small amplitude of the fluctuations suggests that it is not unreasonable to estimate the collisionless damping of the turbulent fluctuations using the damping rates from linear theory. For plasmas with $\beta_i \gtrsim 1$, linear theory shows that the kinetic wave modes corresponding to the fast and slow MHD wave modes are significantly damped collisionlessly at scales $l_{\parallel} \lesssim \lambda_{\text{mfp}i}$ [38, 39, 40]; for lower β_i plasmas, however, the damping is less vigorous. Fast waves are also subject to strong dissipation as they steepen into shocks. Although the argument that these wave modes are damped rapidly in the regime of weak collisionality is plausible, further exploration of this matter is required. For the remainder of this paper, I will neglect the effect of any energy in the fast wave mode (and its kinetic continuation) on the dynamics of the Alfvén, entropy, and slow modes.

In this limit $k_{\parallel} \ll k_{\perp}$ and $\delta B_{\perp} \ll B_0$, the kinetic turbulent dynamics of the Alfvén, slow, and entropy modes is rigorously described by a low-frequency expansion of kinetic theory called gyrokinetics [10, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51]. Exploiting the timescale separation between frequency of the turbulent fluctuations and the ion cyclotron frequency, $\omega \ll \Omega_i$, gyrokinetics averages over the fast cyclotron motion of charged particles in the mean magnetic field. The gyrokinetic approximation orders out the fast MHD wave and the cyclotron resonance, but retains finite-Larmor-radius effects, collisionless damping via the Landau resonance, and collisions. A powerful result from gyrokinetic theory is the conservation of a generalized energy W according to the equation

$$\begin{aligned} \frac{dW}{dt} &= \frac{d}{dt} \int d^3\mathbf{r} \left[\sum_s \int d^3\mathbf{v} \frac{T_{0s} \delta f_s^2}{2F_{0s}} + \frac{|\delta \mathbf{B}|^2}{8\pi} \right] \\ &= \int d^3\mathbf{r} \mathbf{J}_a \cdot \mathbf{E} + \sum_s \int d^3\mathbf{v} \int d^3\mathbf{R}_s \frac{T_{0s} h_s}{F_{0s}} \left(\frac{\partial h_s}{\partial t} \right)_c \end{aligned} \quad (1)$$

where the index s denotes plasma species, δf_s is the perturbation from the Maxwellian equilibrium distribution function F_{0s} with temperature T_{0s} , h_s is the gyrocenter distribution function defined by $\delta f_s = -q_s \phi F_{0s} / T_{0s} + h_s$ with ϕ as the scalar potential, \mathbf{J}_a is an external antenna current driving the system, and $(\dots)_c$ denotes the collision operator [10, 50, 52]. This relation has important implications for the flow of energy in kinetic turbulence, as discussed in detail below.

E. Kinetic Turbulence at $k_{\perp} \rho_i \ll 1$

The dynamics of kinetic turbulence for *all* scales $l_{\parallel} \lesssim \lambda_{\text{mfp}i}$ is described by gyrokinetics as long as the frequency of fluctuations remains below the ion cyclotron frequency [53]; the failure of the gyrokinetic theory when $\omega \rightarrow \Omega_i$ is discussed in Sec III I. Here I consider the properties of this turbulence in the kinetic regime at perpendicular scales larger than the ion Larmor radius, $l_{\perp} \gg \rho_i$ or $k_{\perp} \rho_i \ll 1$. In this regime, Schekochihin *et al.* [10] have demonstrated that the turbulence has the following characteristics: (a) the dynamics of the Alfvén wave cascade decouple from the compressive slow wave and entropy mode fluctuations and are rigorously described by the equations of reduced MHD [20, 24, 54]; (b) the slow and entropy mode dynamics is governed by an ion kinetic equation and these modes are passively mixed nonlinearly by the Alfvén waves; and (c) no energy is transferred between the Alfvénic fluctuations and the compressive fluctuations. Since the Alfvén wave dynamics are given by reduced MHD, the Alfvén waves undergo the same anisotropic cascade described by the theory for strong incompressible MHD turbulence and the cascade continues undamped down to the perpendicular scale of the ion Larmor radius $k_{\perp} \rho_i \sim 1$. Whether the slow and entropy fluctuations are damped in this regime or merely passively mixed by the Alfvén waves with little damping is uncertain at present [10].

F. Kinetic Turbulence at $k_{\perp} \rho_i \sim 1$

At the perpendicular scale of the ion Larmor radius $k_{\perp} \rho_i \sim 1$, two effects come into play. First, the electromagnetic fluctuations may be damped via the Landau resonance with the ions. In the absence of collisions or an external driving mechanism, equation (2) demonstrates that the generalized energy W must be conserved during this process. Through the wave-particle interaction, the electromagnetic fluctuations cause parallel acceleration of the ions, transferring energy from the fields to the ions—the energy lost by the electromagnetic fields is converted into nonthermal structure in velocity space of the ion distribution function.

The second effect is the decoupling of the ion motion from the electromagnetic fluctuations. An ion samples the electromagnetic field over the scale swept out by its fast Larmor motion. For field fluctuations on length scales smaller than the ion Larmor radius $k_{\perp} \rho_i > 1$, this leads to an averaging over the spatially oscillating field; the net field experienced by the particle decreases through the ring-averaging over the Larmor motion [50].

This gradual decoupling of the ions from the fields leads to an inherently nonlinear effect—the entropy cascade. Only recently identified [10], the entropy cascade plays a critical role in the inevitable thermalization of the turbulent energy. Due to the fact that the radius of the Larmor motion for an individual particle depends on the

perpendicular kinetic velocity v_\perp of that particle, particles with high v_\perp decouple from the field more rapidly than those with low v_\perp . The drift velocity of each particle perpendicular to the magnetic field is the $\mathbf{E} \times \mathbf{B}$ velocity; particles with low perpendicular kinetic velocities v_\perp feel a stronger ring-averaged electric field $\langle \mathbf{E} \rangle$ than particles with higher v_\perp , and so drift faster. This differential $\langle \mathbf{E} \rangle \times \mathbf{B}$ drift leads to a coupling of structure in physical space with that in velocity space, resulting in nonlinear phase mixing to smaller scales in both physical and velocity space. This dual cascade is referred to as the *entropy cascade* and occurs for any species s at scales $k_\perp \rho_i \lesssim 1$. As will be explained in Sec II H, this inherently nonlinear phenomenon plays a critical role in the thermalization of turbulent energy.

G. Kinetic Turbulence at $k_\perp \rho_i \gg 1$

Any energy remaining in the electromagnetic fluctuations continues to cascade to scales smaller than the ion Larmor radius $k_\perp \rho_i \gg 1$ as a kinetic Alfvén wave cascade. For many parameter regimes, the damping of the turbulent fluctuations via the Landau resonance with the electrons is often substantial for the entire range of scales $k_\perp \rho_i \gtrsim 1$ [53]. Regardless of the parameter regime, however, by the time the cascade reaches the scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$, the damping always becomes strong enough to terminate the cascade via the Landau resonance with the electrons. Again, in the absence of collisions, the generalized energy W is conserved and the energy lost by the fields builds nonthermal structure in velocity space of the electron distribution function.

H. Thermalization of the Turbulent Energy

As demonstrated by Howes *et al.* [50], damping of the electromagnetic fluctuations by the Landau resonance does not represent heating of the plasma; collisions are required to increase the entropy of the system and realize irreversible plasma heating. The change in entropy S_s can be expressed as

$$T_{0s} \frac{dS_s}{dt} = - \int d^3\mathbf{v} \int \frac{d^3\mathbf{R}_s}{V} \frac{T_{0s} h_s}{F_{0s}} \left(\frac{\partial h_s}{\partial t} \right)_c \quad (2)$$

where V is the plasma volume. It can be shown that the integral is non-positive, so the entropy always increases. Comparing to equation (2), it is clear that, in the absence of driving, the generalized energy may only decrease due to collisions; this collision term represents an increase in entropy and thus represents irreversible heating of the plasma.

The action of the collision operator is to smooth out structure in velocity space of the distribution function; the collision operator is proportional to $\nu \partial^2(h_s/F_{0s})/\partial \xi^2$

[10] where $\xi = v_\parallel/v$ is the pitch angle and ν is a collisional frequency. For weakly collisional plasmas, therefore, the nonthermal structure in velocity space of h_s (created through the parallel acceleration of particles by electromagnetic waves via the Landau resonance) must be driven to sufficiently small scales in velocity space such that $\nu \partial^2(h_s/F_{0s})/\partial \xi^2 \sim \omega$; only then can collisions act rapidly enough to diffuse the fine scale structure in velocity space and increase the entropy. The entropy cascade is therefore the mechanism by which velocity-space structure is driven to very small scales at which point the free energy contained in the nonthermal fluctuations can be thermalized. This ultimate thermalization of the turbulent energy marks the endpoint of the kinetic cascade.

I. Other Considerations

Gyrokinetic theory is only valid for dynamics with characteristic fluctuation frequencies small compared to the ion cyclotron frequency, $\omega \ll \Omega_i$. Since the fluctuation frequency typically increases as the turbulence cascades to smaller scales, it is possible to violate this condition at a scale deep into the kinetic cascade at wavenumbers $k_\perp \rho_i \gg 1$; however, collisionless damping of the turbulent fluctuations can slow this frequency increase along the cascade [53]. Neglecting this damping provides a conservative estimate of the perpendicular wavenumber threshold at which the cyclotron frequency is reached; in this case, the frequency can be estimated throughout the kinetic regime by

$$\frac{\omega}{\Omega_i} \sim \frac{(k_\perp \rho_i)^{2/3}}{\sqrt{\beta_i}} \left(1 + \frac{k_\perp \rho_i}{\sqrt{\beta_i + 2/(1 + T_e/T_i)}} \right)^{2/3} \left(\frac{\rho_i}{L} \right)^{1/3} \quad (3)$$

[53]. The small scale limit of applicability of gyrokinetic theory must be determined on a case by case basis—see Howes *et al.* [53] for a detailed discussion of the limits of validity of gyrokinetics in the solar wind. Astrophysical plasmas in general support an inertial range of many orders or magnitude, so this condition is usually satisfied to wavenumbers beyond the ultimate termination of the kinetic cascade at $k_\perp \rho_e \sim 1$. Taking the specific example of a plasma with $\beta_i = 1$ and $T_i/T_e = 1$, this low-frequency requirement is met when $k_\perp \rho_i \ll (L/\rho_i)^{1/4}$; for a typical astrophysical inertial range of $L/\rho_i \sim 10^8$, this gives $k_\perp \rho_i \ll 100$, so the ion cyclotron frequency is not reached until beyond the scale of $k_\perp \rho_e \sim 1$ (equal to $k_\perp \rho_i \sim 40$).

Next I consider how this picture of the kinetic cascade changes if the cascade is driven at a collisionless scale $L \lesssim \lambda_{\text{mfp}i}$. Since this work is concerned with inertial range turbulence, it is implicitly assumed that the driving scale is much larger than the kinetic scales, $L \gg \rho_i$. Based on the fact that the Alfvén wave cascade, even in collisionless regimes is governed by fluid equations for all scales $k_\perp \rho_i \ll 1$, it seems reasonable to expect that the predictions of strong incompressible MHD turbulence

would continue to hold: an anisotropic cascade will develop leading to an energy spectrum that scales as $k_{\perp}^{-5/3}$ and a scale-dependent anisotropy $k_{\parallel} \propto k_{\perp}^{2/3}$. If the turbulence is driven isotropically at scale L , one expects that gyrokinetic approximation will be satisfied for all scales such that $(k_{\perp} L)^{1/3} \gg 1$ —in other words, the specific predictions for the kinetic cascade outlined here, based on the gyrokinetic theory, will apply beginning at a scale several orders of magnitude below the driving scale.

III. NUMERICAL APPROACH

The efficient numerical study of inertial range turbulence in kinetic plasmas is best conducted using a mixture of computational approaches. At the large, collisional scales, a fluid theory is suitable; a hybrid scheme of fluid electrons and kinetic ions is most efficient at intermediate scales; at the small scales where the turbulent fluctuations are damped and their energy is converted into heat, a kinetic scheme is required for both species. A hierarchy of such schemes is rigorously derived from kinetic theory by Schekochihin *et al.* [10]. In this section, I describe a numerical strategy for the study of the most fascinating aspect of inertial range turbulence in kinetic plasmas—the transition regime from the (reduced) MHD Alfvén wave cascade to the kinetic Alfvén wave cascade at the scale of the ion Larmor radius.

The need for a kinetic treatment of the transition regime limits the spatial dynamic range possible in a numerical simulation. The strategy chosen is to model both ions and electrons gyrokinetically in the neighborhood of $k_{\perp} \rho_i \sim 1$. The simulation results reported here were obtained using *AstroGK*, a new gyrokinetic simulation code designed for the numerical modeling of astrophysical turbulence. The code is based on *GS2*, a mature, widely used gyrokinetic code for the design and interpretation of laboratory experiments in the magnetic fusion program [2, 55]. *AstroGK* is an Eulerian flux tube code with periodic boundary conditions that evolves the five-dimensional distribution function for each plasma species. The spatial components x and y are handled spectrally in Fourier space, and the z -direction is handled with a compact finite differencing scheme. Integration over the velocity-space grids is accomplished using spectral integration by quadrature in energy and pitch angle, while the differentiation needed by the momentum-conserving collision operator in pitch angle is accomplished using finite differencing. The positions of the energy and pitch angle grid points are determined using Legendre polynomials—an example with 128 points in pitch angle and 32 points in energy is shown in Figure 1. The dynamics are fully electromagnetic, evolving the scalar potential ϕ , the parallel component of the vector potential A_{\parallel} , and the parallel component of perturbed magnetic field δB_{\parallel} to describe the fluctuating electromagnetic fields. The linear terms are advanced implicitly to avoid the need to satisfy a Courant con-

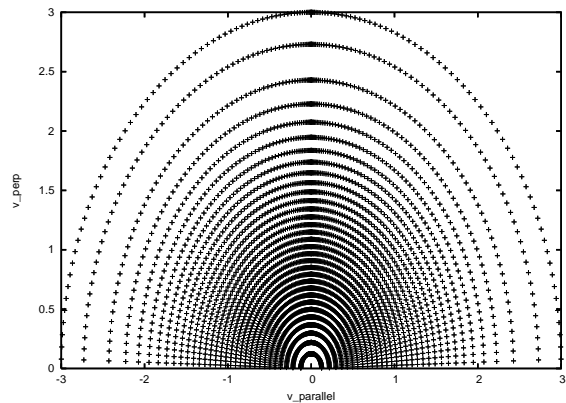


FIG. 1: Plot of grid used in velocity space with 128 pitch angles and 32 energies. Grid point locations are chosen spectrally in a Legendre polynomial basis.

dition for the fast electron dynamics. Nonlinear terms are evaluated by fast Fourier transform in real space and the term is advanced using a 3rd-order Adams-Bashforth scheme. A detailed description of the algorithms used within *AstroGK* is given in Howes *et al.* [56].

Because the range of scales comprising the transition regime falls within the larger kinetic cascade, two developments are essential for the successful modeling of this system: (a) a scheme for modeling the inertial range energy transfer from turbulent fluctuations at scales larger than the simulation domain; and (b) a mechanism for dissipating the turbulence artificially to avoid the unphysical buildup of energy at the smallest resolved scales. Both of these developments will be described in detail and validated in a future work, but here we provide a brief description.

Critical balance posits that the timescale for the nonlinear transfer of energy through the inertial range is of order the linear wave frequency at that scale in the cascade (see Sec II). To simulate this process, the plasma is driven by an external antenna at several large wavelengths comparable to the scale of simulation domain—it is important to excite separately traveling waves moving in both directions along the mean field in order to realize the colliding wave packets necessary for turbulence. The amplitude and phase of each mode determined according to a Langevin equation with a decorrelation time of order the wave period to emulate the conditions within the inertial range of strong MHD turbulence. The antenna drives only a perpendicular fluctuating magnetic field to excite primarily Alfvén waves (although at the amplitudes needed to drive strong turbulence, fluctuations in the direction of the local magnetic field lead to some power coupled into compressive fluctuations).

Although wave-particle interaction through the Landau resonance acts to damp the turbulent cascade, it is often insufficient to terminate the cascade within the resolved range of scales. To prevent a bottleneck of energy at the smallest scales, we act on the turbulence

with a hypercollisional operator—this operator acts on the distribution function with a non-momentum conserving collision operator using a scale-dependent coefficient $\nu_{Hs}(k_{\perp}/k_{\perp\max})^p_s$, where $p_s \in [4, 8]$. The heating caused by hypercollisionality is positive definite; with a carefully chosen coefficient ν_{He} , the electron hypercollisionality is sufficient to terminate the cascade on resolved scales.

Choosing the ideal value of ν_{He} for a given simulation is often difficult. To avoid resorting to trial and error, an adaptive scheme is used to choose and modify the electron hypercollisional coefficient based on nonlinear estimations of the energy transfer frequency and of the effective collisional (including the hypercollisional term) damping rate at a given scale $k_{\perp H\rho_i}$. The energy transfer frequency at $k_{\perp H\rho_i}$ is estimated by $\omega_{nl} \simeq k_{\perp H} \sum |\delta B_{\perp}(k_x, k_y)|^2 / 8\pi$ where the sum is over all Fourier modes (k_x, k_y) within a band of width $\Delta k_{\perp H\rho_i}$ centered at $k_{\perp H\rho_i}$. The nonlinear damping rate is given by the total collisional and hypercollisional heating [50] within the same perpendicular wavenumber band divided by the sum of the total fluctuation energy in the wave band, $\gamma_{nl} \simeq \sum (P_{ci} + P_{ce} + P_{Hci} + P_{Hce}) / \sum E$. The coefficient of electron hypercollisionality ν_{He} is then adjusted within preset bounds in order to achieve a value of $\gamma_{nl}/\omega_{nl} \simeq 1/(2\pi)$ within some tolerance.

Motivated by the desire to mitigate the large computational cost of gyrokinetic simulations of kinetic turbulence, a scheme for reducing the time to reach a statistically steady state of turbulence is used. Initially, a small simulation is run for several eddy turn-around times at the driving scale. The number of cpu-hours required to achieve a steady-state for the small problem is moderate. The restart files from this initial run are processed to add more spatial Fourier modes in the perpendicular direction and finite-difference grid points in the parallel direction—this effectively increases the spatial resolution of the simulation. The initial values of the added Fourier modes are zero and the values at the additional parallel grid points are determined by spline interpolation. When the simulation is restarted, to reach a new steady state, it only needs to run for an eddy turnaround time of the smallest non-zero mode. This Fourier mode expansion technique is used recursively until the desired resolution is reached.

IV. NUMERICAL RESULTS

Here we present results from a simulation of the transition regime from the MHD Alfvén wave cascade to the kinetic Alfvén wave cascade for a plasma with $\beta_i = 1$ and $T_i/T_e = 1$. The spatial dimensions in the plane perpendicular to the mean field are treated pseudospectrally on a 32×32 grid; the parallel direction is treated using a compact finite-difference scheme on 64 grid points. Velocity space resolution uses 128 points in pitch angle and 32 points in energy, for a total of 4096 points over the half-plane $v_{\perp} > 0$ in velocity space; the positions

of grid points are chosen pseudospectrally for maximum accuracy when performing integrations over velocity [56]. The distribution of velocity space grid points for this simulation is shown in Figure 1. A fully ionized proton and electron plasma is specified with a realistic mass ratio of $m_i/m_e = 1836$ and both species are treated gyrokinetically. In summary, the dimensions of the simulation are $(n_x, n_y, n_z, n_{\xi}, n_E, n_s) = (32, 32, 64, 128, 32, 2)$ for a total of 536,870,912 computational mesh points. The simulation used a total of 19,118 cpu-hours on Franklin, the Cray XT4 at NERSC.

A momentum conserving pitch-angle scattering collision operator is employed for like-species collisions with $\nu_i = 0.001$ and $\nu_e = 0.001$; interspecies collisions are neglected. The fully dealiased range of perpendicular wavenumbers covers $k_x \rho_i \in [0.4, 4.0]$ and $k_y \rho_i \in [0.4, 4.0]$.

The simulation is brought to the steady-state using the Fourier mode expansion technique described in Sec III; the problem is run using a spatial grid of $(16, 16, 32)$ for 1.3 outer scale periods before expanding to $(32, 32, 64)$. Zero initial conditions are specified. The Langevin antenna, as described in Sec III, drives the six lowest wavenumber modes corresponding to $(k_x/k_0, k_y/k_0, k_z/k_{z0}) = (1, 0, 1), (0, 1, 1), (-1, 0, 1), (1, 0, -1), (0, 1, -1), (-1, 0, -1)$ with an amplitude of 5.0, a frequency of $\omega = k_{\parallel} v_A$ and a decorrelation frequency of $0.8 k_{\parallel} v_A$.

A fixed ion hypercollisionality is used with $\nu_{Hi} = 0.04$ and $p_i = 8$. Electron hypercollisionality is chosen adaptively to terminate the cascade as described in Sec III. In the initial (pre-expansion) run, the electron hypercollisionality is allowed to adjust within the bounds $10 < \nu_{He} < 1200$ to meet a dissipation criterion of $0.16 < \gamma_{nl}/\omega_{nl} < 0.24$ for all modes $1.6 < k_{\perp} \rho_i < 2.4$; in the expanded restart, the bounds and dissipation criterion are the same, but modes tested fall in the range $3.6 < k_{\perp} \rho_i < 4.4$. The exponent for the electron hypercollisionality is $p_e = 8$.

The high velocity-space resolution simulation presented here was set up to investigate the entropy cascade. Presented in Figure 2 are surface plots and projected contours over velocity space of the real part of the perturbed ion distribution function evolved by **AstroGK**, g_i , defined by

$$g_{i\mathbf{k}} = h_{i\mathbf{k}} - \frac{q_i \langle \phi \rangle_{\mathbf{R}_s \mathbf{k}}}{T_{0i}} F_{0i} - \frac{2v_{\perp}^2}{v_{ti}^2} \frac{\langle \delta B_{\parallel} \rangle_{\mathbf{R}_s \mathbf{k}}}{B_0} F_{0i}. \quad (4)$$

Here the subscript \mathbf{k} denotes a Fourier coefficient and $\langle \phi \rangle_{\mathbf{R}_s \mathbf{k}}$ is the gyro-average at the particle guiding center \mathbf{R}_s . The velocity space structure at the mid-plane in the parallel direction is shown for two Fourier modes $(k_x \rho_i, k_y \rho_i)$. The upper plot shows the mode $(0.4, 0.4)$, corresponding to $k_{\perp} \rho_i \simeq 0.566$; the lower plot $(2.0, 2.0)$, $k_{\perp} \rho_i \simeq 2.83$. Wave-particle interactions via the Landau resonance drive structure in v_{\parallel} in velocity space; the nonlinear phase mixing of the entropy cascade, on the other hand, creates structure in the v_{\perp} direction. The mode at $k_{\perp} \rho_i \simeq 0.566$ in the upper plot shows primarily struc-

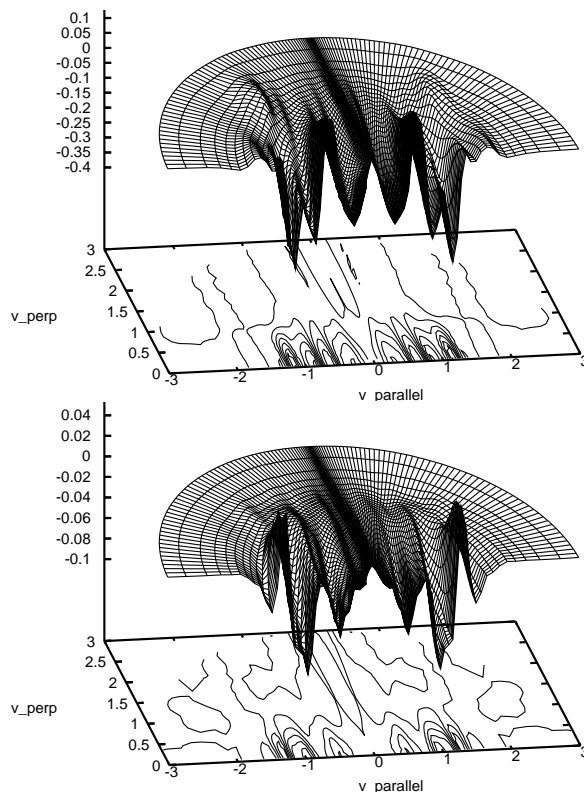


FIG. 2: Surface plots and projected contours of the structure of the real part of the ion distribution function g_s in velocity space for the Fourier modes $(k_x \rho_i, k_y \rho_i) = (0.4, 0.4)$ (upper panel) and $(2.0, 2.0)$ (lower panel) at the mid-plane of the simulation domain in z . The lower plot, at a wavenumber $k_\perp \rho_i \gtrsim 1$, shows v_\perp structure in velocity-space, a finding consistent with the presence of an entropy cascade.

ture in v_\parallel with little variation in v_\perp ; the lower plot, at $k_\perp \rho_i \simeq 2.83$, shows more v_\perp structure in velocity space, a finding consistent with the action of the entropy cascade at $k_\perp \rho_i \gtrsim 1$. Velocity-space in the electron distribution function for these same Fourier modes (not shown) is much smoother, as expected for $k_\perp \rho_e \ll 1$.

Although the results presented here are insufficient to identify clearly the ion entropy cascade, they represent a promising first step. The development of more sophisticated diagnostics of velocity space—for example, taking spectral transforms in velocity space to yield a velocity-space power spectrum—are necessary to explore more thoroughly the dual cascade of energy to small scales in both physical and velocity space. As discussed in Sec II H, this inherently kinetic and nonlinear mechanism plays a key role in the ultimate conversion of turbulent fluctuation energy into heat.

V. APPLICATIONS

Inertial range turbulence arises across broad range of space, astrophysical, and laboratory environments. The

dissipation of the turbulence generally occurs at scales smaller than the ion mean free path; therefore, kinetic plasma physics determines the inevitable plasma heating due to the dissipation of the turbulence. In this section, I will identify a number of systems in which an understanding of the kinetic cascade is necessary to determine the dynamics responsible for the flow of energy and plasma heating. Identification of the characteristic length scales for a specific system—including the driving scale L , the ion mean free path $\lambda_{\text{mfp}i}$, and ion Larmor radius ρ_i —helps to locate its place within the theoretical framework of the kinetic cascade presented in Sec II.

The most accessible turbulent astrophysical plasma is the solar wind; the ability to make *in situ* satellite measurements makes this an invaluable environment for the study of the kinetic cascade. Measurements of turbulence in the solar wind show a $-5/3$ magnetic energy spectrum, interpreted as the beginning of a turbulent inertial range, at scales $L \sim 10^{11}$ to 10^{12} cm [57, 58, 59, 60, 61]. The solar wind is very weakly collisional, with a mean free path of approximately 1 AU, or $\lambda_{\text{mfp}i} \sim 10^{13}$ cm [62, 63]. The ion Larmor radius in the solar wind is $\rho_i \sim 10^6$ to 10^7 cm [53, 64], yielding $\rho_i/L \sim 10^{-5}$ and giving an inertial range over five orders of magnitude in scale. This rather small inertial range, relative most astrophysical contexts, means the threshold at which $\omega \rightarrow \Omega_i$ must be carefully evaluated; a more thorough and quantitative discussion of the applicability of gyrokinetic theory to the turbulent solar wind is contained in Howes *et al.* [53]. Although driven at a collisionless scale $L < \lambda_{\text{mfp}i}$, one expects the general picture of the kinetic cascade described here to apply, as discussed in Sec III. It is important to note, however, that the collisionality is sufficiently weak in the solar wind that the equilibrium particle distribution functions are observed to deviate significantly from Maxwellian [62, 63]. Therefore, velocity-space anisotropy instabilities, such as the mirror and the firehose, may play a role in the transfer of energy outside the scope of this paper, as suggested by satellite observations [65, 66].

Interstellar scintillation due to electron density fluctuations in the interstellar medium (ISM) of our galaxy demonstrate a power law behavior consistent with an exponent of $-5/3$ over twelve orders of magnitude in scale, from $L \sim 10^{20}$ to 10^8 cm [67]. These electron density fluctuations are considered to be passively mixed by the Alfvén wave turbulence in the plasma [14, 16, 33], and thus can be used as a probe of the plasma turbulence. The low end of the range is consistent with the ion Larmor radius in the ISM, $\rho_i \sim 10^8$ cm [68]. The mean free path in the ISM is estimated to be $\lambda_{\text{mfp}i} \sim 10^{11}$ cm. The turbulent ISM therefore ideally fits the picture of inertial range turbulence in kinetic plasmas described in this work.

In the accretion disk around a black hole, the magnetorotational instability (MRI) [69] is able to tap the energy of the gravitational potential to drive turbulence; this turbulence leads to enhanced angular momentum

transport, allowing the plasma to accrete onto the black hole. The MRI injects energy into the turbulent cascade on a scale comparable to the scale height of the thick accretion disk, $L \sim 10^{13}$ cm [70]; this turbulence cascades down to the scale of the ion Larmor radius $\rho_i \sim 10^4$ cm. Due to the predicted high temperatures and low densities of the accretion disk, the mean free path in the plasma is of order the system size, $\lambda_{\text{mfp}i} \sim 10^{13}$ cm. The radiation emitted from the hot, magnetized plasma depends on the black hole properties and the heating of the plasma species by the kinetic dissipation of the turbulence [71, 72, 73]. To understand X-ray observational data from Chandra, the thermalization of the turbulent energy by the kinetic cascade must be characterized.

Although the gradient-driven turbulence studied in the context of fusion plasmas does not develop an inertial range, energetic ions in a burning fusion plasma—arising either from Neutral Beam Injection or as hot α -particles generated by the fusion reaction—can drive shear Alfvén wave instabilities at scales substantially larger than the ion Larmor radius of the thermal plasma [74, 75, 76, 77, 78]. Experimental measurements have confirmed the generation of shear Alfvén modes both by energetic ions [79, 80] and by thermal ions [81]. The wavelength of the Alfvén modes driven by energetic particles is comparable to the size of present-day experiments, so a discrete few Alfvén eigenmodes arise; for the larger scale plasma of the International Tokamak Experimental Reactor (ITER), a broad range of these modes may be driven, possibly leading to the development of a turbulent cascade. Although the primary focus of early studies has been on the confinement of the energetic ions in the presence of these Alfvén wave instabilities, the kinetic cascade driven by energetic ions may prove to be an important channel by which energy from the fusion α -particles is transferred to the kinetic scale of the thermal plasma and converted into heat.

VI. CONCLUSIONS

In this paper, I have presented a general picture of a fundamental plasma physics process: the transfer of turbulent energy through an inertial range from the driving scale to dissipative scales followed by the conversion of this energy into heat. The details of the kinetic cascade of the generalized energy are not yet well understood. Numerical simulations of kinetic turbulence will play a crucial role in illuminating the key elements of the cascade. Here I emphasize several of these important elements:

1. The nonlinear cascade of energy in kinetic plasmas preserves a generalized energy given by equa-

tion (2).

2. Collisions are required to increase the entropy and thus achieve irreversible heating of the plasma.
3. The ultimate thermalization of the turbulent energy is achieved in weakly collisional plasmas through an entropy cascade, a dual cascade of energy to small scales in both physical and velocity space.

I have outlined the numerical approach chosen to investigate turbulence in the transition regime from the Alfvén wave to the kinetic Alfvén wave cascade. The computationally demanding simulations use a Langevin antenna to model driving from larger scales in the inertial range and a hypercollisional operator to remove energy at the smallest resolved scales. Results from a simulation with high velocity-space resolution are consistent with the development of v_\perp structure due to the action of the entropy cascade.

Inertial range turbulence arises in kinetic plasmas across a broad range of space and astrophysical plasmas—including the solar wind, the interstellar medium, and accretion disks around compact objects—and may play an important role in the thermalization of fusion α -particle energy in next-generation burning plasmas.

A program of nonlinear gyrokinetic simulations of kinetic, inertial range turbulence is necessary to explore the key elements of the kinetic cascade described here. The rich plasma physics involved in the inherently kinetic and nonlinear nature of the predicted entropy cascade, along with the computational intensity of the numerical work, make this important physics problem both fun and challenging. Additional aspects of kinetic turbulence must also be examined, such as the effect of strong fast wave turbulence on the Alfvén, slow, and entropy modes; this requires a treatment beyond the scope of gyrokinetics.

Acknowledgments

The author thanks his close collaborators S. Cowley, W. Dorland, G. Hammett, E. Quataert, A. Schekochihin, and T. Tatsuno for contributing significantly to this work. This work was supported by the Department of Energy Center for Multiscale Plasma Dynamics, Fusion Science Center Cooperative Agreement ER54785, by the David and Lucille Packard Foundation, the Aspen Center for Physics, and by the Franklin Early-User Program at National Energy Research Scientific Computing Center.

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